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Confronting electroweak precision measurements with New Physics models

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Abstract. Precision experiments, such as those performed at LEP and SLC, offer us an excellent opportunity to constrain extended gauge model parameters. To this end, it is often assumed that in order to obtain more reliable estimates, one should include the sizable one-loop standard model (SM) corrections, which modify the Z^0 couplings as well as other observables. This conviction is based on the belief that the higher order contributions from the "extension sector" will be numerically small. However, the structure of higher order corrections can be quite different when comparing the SM with its extension; thus one should avoid assumptions which do not take account of such facts. This is the case for all models with $\rho_{\rm tree} \equiv M_W^2/(M_Z^2 \cos^2 \Theta_{\rm W}) \neq 1$. As an example, both the manifest left–right symmetric model and the $SU(2)_{\rm L} \otimes U(1)_Y \otimes \tilde{U}(1)$ model, with an additional Z' boson, are discussed, and special attention to the top contribution to $\Delta\rho$ is given. We conclude that the only sensible way to confront a model with the experimental data is to renormalize it self-consistently. If this is not done, parameters which depend strongly on quantum effects should be left free in fits, though essential physics is lost in this way. We should note that the arguments given here allow us to state that at the level of loop corrections (indirect effects) there is nothing like a "model-independent global analysis" of the data.

1 Introduction

It is a remarkable fact, that precise theoretical predictions of the electroweak SM, obtained after taking into account one-, two-, or even in some cases three-loop effects, fully agree with all experimental data which have been accumulated so far and which have reached a surprisingly high level of precision [1]. Moreover, these theoretical calculations have a high indirect predictive power because of the substantial sensitivity to non-decoupling heavy particle effects. A potentially large top quark contribution to boson self-energies has been recognized a long time ago [2]. Based on this, the top mass has been estimated quite accurately ($m_t^{\rm ind} = 170(184) \pm 7\,{\rm GeV}$, assuming $M_H = M_Z(300\,{\rm GeV})$) [3] prior to its direct determination ($m_t^{\rm dir} = 173.8 \pm 5.2\,{\rm GeV}$) which confirmed the indirect result not so long ago [4]. Now, with the top quark at hand, the only not yet discovered particle which is required in the SM, the Higgs boson, can be studied. At present, the indirect bound after inclusion of the relevant higher order corrections to the Z^0 peak observables implies $m_H < 262\,{\rm GeV}$ at 95% C.L. [5].

It could be that better and better agreement between SM theory and experiments will follow the increasing sophistication of perturbative calculations. In the framework of the SM, this is a natural and obvious possibility.

In the following, let us focus on a different scenario. There are many arguments against the SM to herald in the ultimate theory of elementary particles. We believe that, beyond the SM regime, at higher energies, new physics will show up. Precision experiments provide us an important tool to find its remnants already at present energies. They have been analyzed in the context of many different models, e.g, those which include an additional Z' boson. For details we refer to [6]. It is customary to assume that extended models can be constrained in particular by the neutral current (NC) data, through their modified tree level Z^0 couplings and improved by radiative corrections from the SM. Contributions from the heavy non-standard sector seem to be negligible in a first approximation. However, the situation in general is more complicated and this "standard" approach can be misleading. Before going further we should make this point clearer. In GUT models, typically, per construction a gauge hierarchy exists [7]: a Higgs field exhibits a small vacuum expectation value (VEV) v and determines the SM particle mass spectrum, and another Higgs field with a large VEV V generates the superheavy sector. Decoupling theory states [8] that once a proper identification of the light and of the heavy particles at tree level is done, then such a division will be maintained in any order of perturbative calculations (all the superheavy particle effects enter at most as logarithmic corrections to the light particle effects). However, in phenomenological applications we have no direct experimental access to the parameters of the heavy sector $(V, M_{H_i}, \cdots)$ but only to some effective low energy parameters, like for instance the ρ parameter, which is also a function of the parameters of the heavy sector. If we constrain the low energy effective parameter by experiment (in some physical on-shell renormalization scheme) then we in general set up boundary conditions which are not compatible with the set-up of a gauge hierarchy and the decoupling theory just mentioned does not work. This has further consequences. After letting the superheavy masses go to infinity, the low energy effective theory (assuming light fields are the same as in the SM) is not any longer renormalizable, much in the same way as the low energy effective four fermion interactions are non-renormalizable if we fix G_μ and let $M_W \to \infty$ (which requires $g \to \infty$ simultaneously).

2 Discussion

To outline our point of view let us consider left (L)–right (R) symmetric models (LRM) with gauge group $SU(2)_{\mathrm{L}} \otimes SU(2)_{\mathrm{R}} \otimes U(1)_{B-L}$ which are manifestly LR symmetric before the symmetry is broken by the appropriate Higgs mechanism [9]. These models have all the necessary features of a large class of extended models, and some results at the one-loop level have lately been obtained [10, 11] which are applicable to LEP/SLC physics.

Let us start by considering the Z^0 partial decay widths and forward–backward asymmetries, theoretically described by the following relations [12]:

$$\begin{aligned}\Gamma_{f\bar{f}} &= \frac{N_c^f G_{\mathrm{F}} M_Z^3}{6\pi\sqrt{2}}\beta \\ &\quad\times\left(\frac{3-\beta^2}{2}v_f^2+\beta^2 a_f^2\right)K_{\mathrm{QCD}}K_{\mathrm{QED}}, \qquad (1)\\ A_{\mathrm{FB}}^f &= \frac{3}{4}A_e A_f, \quad A_f=\frac{2v_f a_f}{\left(v_f^2+a_f^2\right)}, \qquad (2)\end{aligned}$$

where N_c^f is the color factor, β the fermion velocity, the K factors take into account electromagnetic and strong corrections, and v_f and a_f are vector and axial fermion couplings. In the LRM model these can be written in the simple and compact form (T_f^3, Q_f being the fermion isospin and charge, respectively)

$$\begin{aligned}v_f &= \sqrt{\rho_{\mathrm{eff}}^f}(T_f^3 - 2Q_f \sin^2\Theta_{\mathrm{eff}}^f)\\ &\quad\times(\cos\phi - \sin\phi/\sqrt{\cos 2\Theta_{\mathrm{W}}}), \qquad (3)\\ a_f &= \sqrt{\rho_{\mathrm{eff}}^f}T_f^3(\cos\phi+\sin\phi\sqrt{\cos 2\Theta_{\mathrm{W}}}). \qquad (4)\end{aligned}$$

Here ϕ is the Z^0–Z' mixing angle and the two other angles are connected to the effective weak mixing parameter $\sin^2\Theta_{\mathrm{eff}}^f$ in the NC at the Z^0 resonance (for which (3) is the defining equation) and the weak mixing angle Θ_{W} defined via the vector boson masses by

$$\sin^2\Theta_{\mathrm{W}} = 1-\frac{M_W^2}{\rho_0 M_Z^2}. \qquad (5)$$

While the ρ parameter is unity at the tree level in the SM, it differs from unity in many extended models: $\rho_{\mathrm{tree}} \equiv \rho_0 \neq 1$. Let us assume that higher order effects are small, actually, and that they can be gathered by SM like the relations

$$\begin{aligned}\rho &= \frac{\rho_0}{(1-\Delta\rho)},\\ \rho_{\mathrm{eff}}^f &= \rho(1+\Delta\rho_{\mathrm{rem}}^f). \qquad (6)\end{aligned}$$

In the LR model ρ_0 should be understood as $\rho_0/\rho^{\pm}$ where ρ_0 is given by $\rho_0 = 1+\sin^2\phi\left(M_{Z_2}^2/M_{Z_1}^2-1\right)$ and is due to the Z–Z' mixing and $\rho^{\pm} = 1+\sin^2\phi_{\pm}(M_{W_2}^2/M_{W_1}^2-1)$ is due to the W–W' mixing.

In terms of the input parameters α, G_{F}, $M_Z, \cdots$ with $A_Z = (\pi\alpha(M_Z))/((2)^{1/2}G_{\mathrm{F}})$ and $\alpha(M_Z) = \alpha/(1-\Delta\alpha)$ we can predict

$$\sin^2\Theta_{\mathrm{W}} = \frac{1}{2}\left[1-\sqrt{1-\frac{4A_Z}{\rho M_Z^2}(1+\Delta r_{\mathrm{rem}})}\right], \qquad (7)$$

$$\sin^2\Theta_{\mathrm{eff}}^f = \frac{1}{2}\left[1-\sqrt{1-\frac{4A_Z}{\rho_{\mathrm{eff}}^f M_Z^2}\left(1+\Delta r_{\mathrm{rem}}^f\right)}\right], \qquad (8)$$

with leading higher order corrections incorporated in resummed form [13]. Let us put $\phi = 0$, so that pure SM physics is restored. Then the terms $\Delta\alpha$, $\Delta\rho$, $\Delta\rho_{\mathrm{rem}}^f$, Δr_{rem}, $\Delta r_{\mathrm{rem}}^f$ include SM radiative corrections to the Z^0 and muon physics [12]. These depend on many details, for instance, the f superscript means that actually $\sin^2\Theta_{\mathrm{eff}}^f$ and ρ_{eff}^f are not universal quantities but are different for each fermion flavor produced at the Z^0 resonance through flavor specific vertex (and box) effects. The flavor dependence, however, is relatively small except for $f = b$ which requires a separate treatment. Referring to lepton universality, we denote the leptonic weak mixing parameter by $\sin^2\Theta_{\mathrm{eff}}^\ell$ ($\ell = e, \mu$ or τ). Some of the radiative corrections are dominant. For instance, in (7) the two leading effects have been incorporated by including the running of the fine structure constant (shift by $\Delta\alpha$) from low to high (Z mass) energies and the renormalization of $\rho_0 = 1$ by the large mass splitting between top and bottom quarks in boson self-energies (shift by $\Delta\rho$):

$$\Delta\rho = \Delta\rho^{\mathrm{top}} + \Delta\rho_{\mathrm{rem}}, \quad \Delta\rho^{\mathrm{top}} = 3x_t, \quad x_t \equiv \frac{\sqrt{2}G_{\mathrm{F}}}{16\pi^2}m_t^2. \qquad (9)$$

For $f \neq b$, all other contributions indexed by “rem” are smaller remainder terms, e.g., $\Delta r_{\mathrm{rem}}^f$ is the remainder gathering non-leading effects from boson self-energies, vertices and boxes. In the case $f = b$ there is a leading top mass correction coming from the $Zb\bar{b}$ vertex [14] which can be incorporated as

$$\rho^b_{\text{eff}} = \rho^\ell_{\text{eff}}(1+\tau_b)^2, \tag{10}$$

$$\sin^2\Theta^b_{\text{eff}} = \sin^2\Theta^\ell_{\text{eff}}/(1+\tau_b), \tag{11}$$

with $\tau_b = -2x_t$ (see (9)). All correction factors influence $\Gamma_{f\bar{f}}$, A^f_{FB} given in (1) and (2), as well as other observables.

Now, let us switch on "new physics" again ($\phi \neq 0$). The question is (apart from coupling modifications) what is going to change in the loop effects. As written in the Introduction, the "canonical" answer is [15] (here we refer only to papers where LRM have been considered): (6)–(9) will not be changed, except for negligible contributions affecting the sub-leading terms. The leading behavior will be governed by the SM.

However, beyond the tree level, as shown in [11], a substantial part of the relevant radiative corrections change completely, and there is only a weak relationship between the radiative corrections of the SM and the new physics model (NPM = extended SM). While corrections like $\Delta\alpha$ are universal, others may change dramatically, in particular the non-decoupling heavy particle effects. For instance, one of the most important one-loop terms, $\Delta\rho^{\text{top}}$ loses its m_t^2 dependence, namely, in the LR model we obtain

$$\Delta\rho^{\text{top}}_{\text{LR}} = \frac{\sqrt{2}G_{\text{F}}}{8\pi^2} c_W^2 \left(\frac{c_W^2}{s_W^2} - 1\right) \frac{M^2_{W_1}}{M^2_{W_2} - M^2_{W_1}} 3m_t^2 \tag{12}$$

as a leading term. For a W_2 boson mass of the order of 400 GeV or larger this contribution is much smaller than the SM one, actually even smaller than the SM logarithmic terms. Besides this, other particles like heavy neutrinos and heavy scalars [10,11] influence substantially the sub-leading terms in (6)–(9).

The traditional philosophy simply breaks down. When fitting parameters within the framework of a NPM, e.g. the Z^0–Z' mixing parameter ϕ, the only way of including one-loop effects is to renormalize the whole model. Except from universal corrections like the QED shift $\Delta\alpha$, it is not legitimate to use radiative corrections from the SM for its extension unless ρ_0 remains unity. Affected are in particular the zero momentum gauge boson contributions. Although at low energies and at tree level the LRM seems to be effectively equivalent to the SM ($\phi, \phi_\pm \to 0$ and $M_{Z_2}, M_{W_2} \to \infty$), radiative corrections can be quite different and do not follow this naive expectation (see (12) and $M_{W_2} \to \infty$)[1].

The crucial point is that associated with the additional free parameters there are new divergences and hence new subtractions needed. Then (6)–(9) will get additional contributions and now will be functions of the extended set of input parameters (SM parameters plus $\phi, M_{W_2}, M_{Z_2}, \cdots$). Let us note that the naively written one-loop level definition of $\sin^2\Theta^f_{\text{eff}}$ in (8) should also be different from its SM structure. The LRM angle ϕ can be fixed at tree level by $[F(\Theta_{\text{W}}) = -\cos^2\Theta_{\text{W}}/(\cos 2\Theta_{\text{W}})^{1/2}]$

$$\sin 2\phi = \frac{\left[\left(g^2+g'^2\right)\left(M^2_{W_2}+M^2_{W_1}\right) - \frac{1}{2}g^2\left(M^2_{Z_1}+M^2_{Z_2}\right)\right]}{F(\Theta_{\text{W}})\left(M^2_{Z_2}-M^2_{Z_1}\right)\left(\frac{1}{2}g^2+g'^2\right)}, \tag{13}$$

and extraction of $\sin^2\Theta^f_{\text{eff}}$ from the Z fermion couplings (3) at the one-loop level will also include its renormalization. The same touches on the $\sin^2\Theta_{\text{W}}$ definition (5) and (7), where $\rho_{\text{tree}} \neq 1$ (see [11] for the renormalization of the $\sin^2\Theta_{\text{W}}$ parameter).

The observation that the structure of higher order effects is highly model dependent was pointed out long time ago in [16] for the case of models with an enhanced Higgs sector (the so-called "unconstrained" extended models) for which the custodial symmetry exhibited by the SM Higgs is violated at the tree level, causing $\rho_{\text{tree}} \neq 1$. In [17] it was shown in general how the SM radiative corrections are modified in models which require a direct or indirect renormalization of the ρ parameter. See [18] for an analysis of precision observables in a SM enhanced by an additional Higgs triplet. In any case, if ρ is itself a free parameter or a function of other input parameters, the quadratic top mass contributions coming from self-energy diagrams are lost by the required subtraction and only logarithmic top mass dependences remain. The dependence on the Higgs mass is also affected substantially (see the Appendix for details). Hence, in models with $\rho_{\text{tree}} \neq 1$ the LEP/SLC indirect top mass limits become obsolete. Such models are unable to explain why the direct top mass agrees with the one obtained from precision measurements of the loop effects in $\Delta\rho$. The coincidence $m_t^{\text{ind}} \simeq m_t^{\text{dir}}$ obtained by SM fits has meaning only when ρ is a finite calculable quantity, which requires $\rho_{\text{tree}} = 1$, like in the SM or in its minimal supersymmetric extension. In the case of the LRM, which we have discussed before, the phenomenon of a complete change in the large m_t behavior of (12) was obtained in a different renormalization scheme which did not treat ρ_{tree} itself as an independent parameter. In contrast to the m_t^2 dependence originating in the W and Z self-energies at zero momentum, the m_t^2 dependence of the $Zb\bar{b}$ vertex is not (or little) affected when going to an extended model. Therefore, the observables including b quark contributions, like $\Gamma_{b\bar{b}}$, A^b_{FB}, the Z width or the Z peak cross-section, still exhibit strong m_t dependences (now very different from the ones in the SM) which allow one to get good indirect m_t bounds [18,19]. However, there is no good reason why the new bounds should coincide with the ones obtained in the SM. This does not necessarily mean that one cannot obtain equally good global fits, because in the extended model more free parameters are at our disposal (ρ_0 free fits [19]).

The mentioned "instability of quantum effects" may also be observed in rather simple modifications of the SM, like, the $SU(2)_{\text{L}} \otimes U(1)_Y \otimes \tilde{U}(1)$ models, which often arise as the low energy limit of interesting GUT's, and which exhibit an additional Z' boson mixing with the Z^0. We may restrict ourselves to consider the *constrained* version, where Higgs bosons transform as doublets or singlets of $SU(2)_{\text{L}}$. Aspects of the renormalization of such models have been considered in [20].

[1] As discussed at the end of the Introduction we should be careful in referring to decoupling in the limit $M_{W_2} \to \infty$. In reality we fix $\Delta\rho^{\text{top}}_{\text{LR}}$ to experimental data, which means also that a limit $M_{W_2} \to \infty$ not necessarily is allowed any longer.

If Z' mixes with Z^0 then we obtain neutral vector bosons of masses $M_{Z_1}(\leq M_{Z^0})$ and $M_{Z_2}(\geq M_{Z^0})$ and at the tree level the Z_1–Z_2 mixing angle ϕ is fixed by

$$\tan^2\phi = \frac{M_W^2/\cos^2\Theta_{\mathrm{W}} - M_{Z_1}^2}{M_{Z_2}^2 - M_W^2/\cos^2\Theta_{\mathrm{W}}}, \tag{14}$$

or, equivalently,

$$\rho_{\mathrm{tree}} \equiv \rho_0 \equiv \frac{M_W^2}{M_{Z_1}^2\cos^2\Theta_{\mathrm{W}}} = 1 + \sin^2\phi\left(\frac{M_{Z_2}^2}{M_{Z_1}^2} - 1\right) > 1. \tag{15}$$

In [20] $\sin^2\Theta_{\mathrm{W}}$ has been calculated in terms of α, G_{F} and M_W at one-loop order,

$$\sin^2\Theta_{\mathrm{W}} = \frac{\pi\alpha}{\sqrt{2}G_{\mathrm{F}}M_W^2}(1+\Delta\tilde{r}), \tag{16}$$

with the conclusion that $\Delta\tilde{r} \simeq \Delta r^{\mathrm{SM}}$ up to negligible corrections, in a scheme where continuity in the limit $\phi \to 0$ is imposed by hand. Note that this relation, which derives from the charged current (CC) muon decay, is not modified at the tree level. Thus $\sin^2\Theta_{\mathrm{W}} \simeq \sin^2\Theta_{\mathrm{W}}^{\mathrm{SM}}$ when calculated in terms of α, G_μ, M_W and the subtraction is imposed at $\phi = 0$.

However, if we calculate $\sin^2\Theta_{\mathrm{W}}$ in terms of α, G_{F} and M_Z (the standard input parameters for precision calculations), again at one-loop order, we have

$$\sin^2\Theta_{\mathrm{W}}\cos^2\Theta_{\mathrm{W}} = \frac{\pi\alpha}{\sqrt{2}G_{\mathrm{F}}\rho_0 M_Z^2}(1+\Delta\tilde{r}), \tag{17}$$

which is modified by the appearance of the new parameter ρ_0, which has to be renormalized now as well. Since ρ_0 acts as a free parameter we cannot any longer get the m_t bounds of the SM. In the commonly accepted procedure one would argue as follows: in a linear approximation, due to $\rho_0 = 1 + \Delta\rho_0 \neq 1$ we effectively get an extra classical contribution

$$\delta\Delta r = -\frac{\cos^2\Theta_{\mathrm{W}}^0}{\sin^2\Theta_{\mathrm{W}}^0}\Delta\rho_0, \Delta\rho_0 = \sin^2\phi\left(\frac{M_{Z_2}^2}{M_{Z_1}^2} - 1\right), \tag{18}$$

where

$$\sin^2\Theta_W^0 = 1 - \frac{M_W^2}{M_Z^2}. \tag{19}$$

Thus it looks as if we would substitute in (6)

$$\Delta\rho^{\mathrm{top}} \to \Delta\rho^{\mathrm{top}} + \Delta\rho_0, \tag{20}$$

with both contributions positive. Formally, one seems to be able to constrain both m_t and ρ_0. After a full one-loop renormalization of the NPM a term $\Delta\rho^{\mathrm{top}} \sim m_t^2$ is absent, however, and the conventional recipe breaks down (see the Appendix for details).

We conclude that self-consistent constraints on the NPM parameters can be obtained only by a consequent order by order analysis of the model.

The question remains: what can we say about constraints on the additional parameters of the NPM without knowledge of the higher order corrections? As mentioned earlier, parameters which receive substantial model-dependent contributions from radiative corrections have to be treated as free parameters now, with the consequence that important information obtained from SM fits gets lost, e.g. the top mass prediction.

Let us take the LEP/SLC data [3]

$$\begin{aligned} M_Z &= 91.1867 \pm 0.0021\,\mathrm{GeV}, \\ \Gamma_Z &= 2.4939 \pm 0.0024\,\mathrm{GeV}, \\ \sigma_h^0 &= 41.491 \pm 0.058\,\mathrm{nb}, \\ R_\ell &= 20.765 \pm 0.026, \\ A_{FB}^{0,\ell} &= 0.01683 \pm 0.00096. \end{aligned}$$

They have been extracted from the line-shape and lepton asymmetries. We will also use A_ℓ, R_b^0, R_c^0, $A_{FB}^{0,b}$, $A_{FB}^{0,c}$, A_b, A_c (values, correlation matrices and definitions are to be found in [3]). The important point is that all of them are expressible through (1)–(4).

According to our approach $\sin^2\Theta_{\mathrm{eff}}^f$ and ρ_{eff}^f should be left as free parameters [2]. Then $\sin^2\Theta_{\mathrm{W}}$ is given by (5) with $\rho_0 = \rho_{\mathrm{eff}}^f$ (we consistently use tree level relations). We know that physics connected to the b quark differ from those of other fermions. We take for $\sin^2\Theta_{\mathrm{eff}}^b$ relation (11) but leave ρ_{eff}^b as a free parameter. Later we will use relation (10) to get a bound on m_t via the Zbb vertex which yields a main source of information on the top mass in fits with free $\sin^2\Theta_{\mathrm{eff}}^\ell$.

To sum up, we have 18 physical data (M_Z, Γ_Z, σ_h^0, R_ℓ, $A_{FB}^{0,\ell}$, A_ℓ, R_b^0, R_c^0, $A_{FB}^{0,b}$, $A_{FB}^{0,c}$, A_b, A_c, $\sin^2\Theta_{\mathrm{eff}}^\ell$, $\sin^2\Theta_{\mathrm{eff}}^b$, $\sin^2\Theta_{\mathrm{W}}$, m_t, α_s, M_W) parametrized through v_f and a_f ((3) and (4)). The latter are functions themselves of the four completely free parameters ρ_{eff}^ℓ, ρ_{eff}^b,ϕ, $\sin^2\Theta_{\mathrm{eff}}^\ell$ and useful constraints are obtained only because a_f and v_f differ for various fermion flavors.

The χ^2 minimization procedure gives [21] (at 90 % C.L.)

$$|\phi| \leq 0.003, \tag{21}$$

$$\rho_{\mathrm{eff}}^\ell = 1.005 \pm 0.004, \tag{22}$$

$$\rho_{\mathrm{eff}}^b = 0.998 \pm 0.009, \tag{23}$$

and

$$\sin^2\Theta_{\mathrm{eff}}^\ell = 0.232 \pm 0.001. \tag{24}$$

If we assume, as already discussed, that the $Zb\bar{b}$ vertex is not affected too much by the new physics then the relations given in (10) and (11) hold and an upper limit on the

[2] Note that $\sin^2\Theta_{\mathrm{eff}}^\ell(Q_{FB})$ extracted from hadronic charge asymmetry $< Q_{FB} >$ by the LEP Collaborations (see [3] and references therein) relies in an essential way on the SM. In contrast to common practice we do not use this observable in a derivation of NPM constraints. This fact is often ignored in analyses.

top mass can be derived. We get within the given errors from (22) and (23)

$$\frac{\rho^b_{\text{eff}}}{\rho^e_{\text{eff}}} = (1+\tau_b)^2 \geq 0.98, \tag{25}$$

from which $m_t \leq 225$ GeV follows. See also the discussion in [19].

In the framework of the LR model, for $M_{Z_2} >> M_{Z_1}$ we may use the approximate relation [22]

$$\phi \simeq \sqrt{2\cos\Theta_{\text{W}}}\frac{M^2_{Z_1}}{M^2_{Z_2}} \tag{26}$$

in order to obtain the Z_2 mass bound

$$M_{Z_2} \geq 1420\text{GeV}. \tag{27}$$

This is a quite strong constraint [3] (see [25] for a comprehensive analysis including also the low energy data). However, we should stress here that treating $\sin^2\Theta_{\text{W}}$ and $\sin^2\Theta^f_{\text{eff}}$ as "black boxes" we lost essential physical information on the NPM. In reality, at loop level, $\sin^2\Theta_{\text{W}}$ and $\sin^2\Theta^f_{\text{eff}}$ are complicated functions of new parameters e.g. ϕ, M_{Z_2}, M_{W_2}, heavy neutrinos, extra Higgs particles. We do not know what the relation is between the result obtained in (21)–(24) and those which would come from the full one-loop analysis.

3 Conclusions

To summarize, fitting precision data requires precise predictions (including the relevant higher order effects) to be confronted with the data, i.e., for conclusive comparisons the precision of data and theory have to match as far as possible. For example, fitting the electroweak data with SM tree level predictions only would rule out the SM, while including radiative corrections leads to perfect agreement. These rules apply as well for any extension of the SM. Such NPM exhibit additional free parameters, so that parameters of the SM, which may be substantially shifted by higher order SM corrections, turn into free parameters in the NPM. It is thus obvious that taking into account just the SM radiative corrections plus the tree level extension cannot make sense in general. This is the case in particular for all $\rho_{\text{tree}} \neq 1$ extensions. In our opinion, there are much more model dependences of global fits and their interpretation than usually presumed. As an example, the S, T, U parameter description of physics beyond the SM [26,27] directly only applies to $\rho = 1$ extensions, like models with additional fermion families (already discussed in [2]), additional scalar singlets and doublets, massive neutrinos which might exhibit ν mixing and supersymmetric extensions of the SM. For $\rho \neq 1$ extensions our discussion concerning $\Delta\rho$ and the m_t bounds applies directly to T which is defined as $\Delta\rho/\alpha$. S and U are scale sensitive quantities which are *expected* to survive modifications in the renormalization procedure. The problem here is that the gauge boson self-energies which are intended to be described by these parameters are not observables themselves. They cannot be separated in general from vertex and box corrections. See also the discussion within the effective Lagrangian approach [28] for this point. One of the most important results of the electroweak precision measurements is the fact that ρ is very close to its SM prediction. All models with $\rho_0 \neq 1$ have a severe fine tuning problem: why does the value of the "ρ_0 free" fits yield a result which by accident is very close to the SM prediction?

[3] Our analysis should be taken as an illustration only. Our approach is not fully self-consistent. Some of the Z^0 parameters used here are so-called pseudo-observables, which have been extracted from experimental data utilizing SM radiative corrections [23]. We could in principle extract the Z^0 parameters from experimental data using the ZFITTER program [24] leaving, according to our approach, model-dependent radiative corrections as free numbers and see precisely the difference in the fits. Also γ–Z interference should be taken into account in an appropriate manner.

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Appendix: Modification of the SM top quark and Higgs boson contributions in extensions of the SM with $\rho \neq 1$

One of the crucial features of the SM is the validity of the relationship

$$\rho = \frac{G_{NC}}{G_{CC}} = \frac{M^2_W}{M^2_Z\cos^2\Theta^0_{\text{W}}} = 1,$$
$$\cos^2\Theta^0_{\text{W}} = \frac{g^2}{g^2+g'^2}, \tag{28}$$

at the tree level. As discussed in the main text, many extensions of the minimal SM share this property with the SM. For all these models

$$\frac{G_{NC}}{G_{CC}}(0) = \rho = \frac{1}{1-\Delta\rho} \tag{29}$$

is a calculable quantity which is sensitive to weak hypercharge breaking and weak isospin breaking due to mass splittings of multiplets. Here we mention that if $\rho_0 = \rho_{\text{tree}} \neq 1$ one should consequently replace

$$\sin^2\Theta^0_{\text{W}} \to \sin^2\Theta_W = (e/g)^2 = 1 - \frac{M^2_W}{\rho_0 M^2_Z}, \tag{30}$$
$$\Delta r \to \Delta r_g = 1 - \frac{\pi\alpha}{\sqrt{2}G_\mu M^2_W}\frac{1}{\sin^2\Theta_{\text{W}}},$$

in all SM formulae. If we define $\Delta\rho_0$ in analogy to (29) by

$$\rho_0 = \frac{1}{1-\Delta\rho_0},$$

we have

$$\sin^2\Theta_W = \sin^2\Theta_{\mathrm{W}}^0 \left(1 + \frac{\cos^2\Theta_{\mathrm{W}}^0}{\sin^2\Theta_{\mathrm{W}}^0}\Delta\rho_0\right),$$

and hence the *exact* relation

$$\frac{1}{1-\Delta r} = \frac{1}{1-\Delta r_g}\left(1 + \frac{\cos^2\Theta_{\mathrm{W}}^0}{\sin^2\Theta_{\mathrm{W}}^0}\Delta\rho_0\right) \tag{31}$$

holds. The experimental bounds mentioned before suggest that deviations from $\rho_0 = 1$ can be treated as perturbations. In the standard approach such "tree level" perturbations may be included by using

$$(\Delta\rho)_{\mathrm{irr}} \to (\Delta\rho)_{\mathrm{irr}} + (1-\rho_0^{-1}), \tag{32}$$

or, in linear approximation, simply by adding

$$\delta\Delta r = -\frac{\cos^2\Theta_{\mathrm{W}}^0}{\sin^2\Theta_{\mathrm{W}}^0}\Delta\rho_0, \tag{33}$$

where $\Delta\rho_0$ depends on the extension considered. This approach is wrong, however. In the following we show which of the SM contributions survive once ρ_0 is subject to renormalization.

Consider the low energy effective neutral current "Fermi constant"

$$\sqrt{2}G_{\mathrm{NC}} = \frac{\pi\alpha}{M_Z^2\cos^2\Theta_{\mathrm{eff}}^\ell\sin^2\Theta_{\mathrm{eff}}^\ell}\left(1+\delta_{\mathrm{NC}}\right). \tag{34}$$

Since it is an independent parameter here and hence appears subtracted independently of $G_{\mathrm{CC}} = G_\mu$, no term $\Delta\rho$ is left over and we have[4] ($s_W^2 = 1 - c_W^2$, $c_W^2 = M_W^2/M_Z^2$)

$$\delta_{\mathrm{NC}} = \Delta\alpha - \frac{1}{c_W^2}\Delta_1 + \delta_{\mathrm{NC}}^{\mathrm{vertex+box}}. \tag{35}$$

For the leading heavy particle effects we obtain

$$\delta_{\mathrm{NC}}^{\mathrm{top}} = -K\frac{2}{3c_W^2}\ln\frac{m_t^2}{M_Z^2},$$
$$\delta_{\mathrm{NC}}^{\mathrm{Higgs}} = -K\frac{1}{3c_W^2}\left(\ln\frac{m_H^2}{M_Z^2} - \frac{5}{3}\right), \tag{36}$$

where $K = \alpha/(4\pi s_W^2)$. For the charged current amplitude we have

$$\sqrt{2}G_\mu = \frac{\pi\alpha}{M_W^2\sin^2\Theta_{\mathrm{eff}}^\ell}\left(1+\delta_{\mathrm{CC}}\right), \tag{37}$$

where α and M_W are renormalized as usual and $\sin^2\Theta_{\mathrm{eff}}^\ell$ as in the NC case. With G_μ fixed from the μ decay rate we have

$$\delta_{\mathrm{CC}} = \Delta\alpha - \Delta_1 + \Delta_2 + \delta_{\mathrm{CC}}^{\mathrm{vertex+box}}. \tag{38}$$

[4] In the notation of [27] $\Delta\rho = \varepsilon_1$, $\Delta_1 = \varepsilon_3$ and $\Delta_2 = \varepsilon_2$, which up to normalization correspond to T, S and U [26].

The leading heavy particle effects in this case are

$$\delta_{\mathrm{CC}}^{\mathrm{top}} = K\frac{4}{3}\ln\frac{m_t^2}{M_W^2},$$
$$\delta_{\mathrm{CC}}^{\mathrm{Higgs}} = K\frac{1}{3}\left(\ln\frac{m_H^2}{M_W^2} - \frac{5}{3}\right). \tag{39}$$

For the ratio we find

$$\rho = \frac{G_{\mathrm{NC}}}{G_\mu} = \frac{M_W^2}{M_Z^2\sin^2\Theta_{\mathrm{eff}}^\ell}\left(1-\Delta\hat{\rho}'\right), \tag{40}$$

where $\Delta\hat{\rho}' = \delta_{\mathrm{CC}} - \delta_{\mathrm{NC}}$. Here the leading heavy particle terms read

$$\Delta\hat{\rho}'^{\,\mathrm{top}} = K\left(\frac{4}{3} + \frac{2}{3c_W^2}\right)\ln\frac{m_t^2}{M_W^2},$$
$$\Delta\hat{\rho}'^{\,\mathrm{Higgs}} = -K\frac{1}{3}\frac{s_W^2}{c_W^2}\left(\ln\frac{m_H^2}{M_W^2} - \frac{5}{3}\right). \tag{41}$$

Obviously no terms proportional to m_t^2 (which originate in the SM from the W and Z self-energies at zero momentum) have survived and the leading heavy Higgs terms are reduced by roughly a factor 10 (!) relative to the minimal SM. In contrast, the m_t^2 terms showing up for the $Zb\bar{b}$ vertex and the observables which depend on it are at most weakly affected due to mixing effects.

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